The golden mean as clock cycle of brain waves

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Abstract

The principle of information coding by the brain seems to be based on the golden mean. Since decades psychologists have claimed memory span to be the missing link between psychometric intelligence and cognition. By applying Bose–Einstein-statistics to learning experiments, Pascual-Leone obtained a fit between predicted and tested span. Multiplying span by mental speed (bits processed per unit time) and using the entropy formula for bosons, we obtain the same result. If we understand span as the quantum number n of a harmonic oscillator, we obtain this result from the EEG. The metric of brain waves can always be understood as a superposition of n harmonics times 2Φ, where half of the fundamental is the golden mean Φ ($= 1.618$) as the point of resonance. Such wave packets scaled in powers of the golden mean have to be understood as numbers with directions, where bifurcations occur at the edge of chaos, i.e. $2\Phi = 3 + \phi^3$. Similarities with El Naschie's theory for high energy particle's physics are also discussed.
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1. Introduction

"It bothers me that, according to the laws as we understand them today, it takes ... an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what a tiny piece of space–time is going to do? So I have often made the hypothesis that ultimately physics will not require a mathematical statement, that in the end the machinery will be revealed and the laws will turn out to be simple, like the checker board with all its apparent complexities,"

wrote Feynman in 1965 (cited from [1, p. 638]. Wolfram [2], too, believes that there are quite simple mechanisms that underlie human reasoning. He asserts that the use of memory is what in fact underlies almost every aspect of human thinking. Capabilities like generalization, analogy and intuition immediately seem very closely related to the ability to retrieve data from memory on the basis of similarity.

Already in 1966, Kac [3] had put forward the question: Can one hear the shape of a drum? In order to find an answer, Kac asks for the energy in the frequency interval df. To this end, he calculates the number of harmonics which lie between the frequencies f and df and multiplies this number by the energy which belongs to the frequency f, and which according to the theory of quantum mechanics is the same for all frequencies. By solving the eigenvalue problem of the wave equation, Kac is able to state that one can not only hear the area of a reflecting surface, its volume and circumference, but also the connectivity of paths of an irregular shaped network. If the brain waves had the possibility to measure and hence to know the eigenvalues of a spatially distributed information amount, they would have nearly

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36 perfect access to information and—in terms of communication theory—perform nearly perfect bandlimited processing.
37 As we know, the eigenvalues are proportional to the squares (i.e. variances) of resonant frequencies [4].
38   The question whether brain waves reflect underlying information processing is as old as EEG research itself.
39 Therefore, relationships between well-confirmed psychometric and psychophysiological empirical facts [5] and EEG
40 spectral density are very interesting.

41 **2. Memory span as the quantum of action of thought**

42   Ever since attention became the object of scientific study, psychologists have recognised that it possesses a quan-
43 titative dimension in terms of the maximum number of items to which a person can attend at one time. It now seems
44 almost universally accepted [6] that short-term memory has a capacity limit of 7 ± 2 [7]. The possibility that such
45 quantitative limits on attention span might be related to qualitative differences in thought and reasoning was recognised
46 by Piaget [8]. Beginning with Pascual-Leone [9], the prediction of children's reasoning from estimates of their memory
47 span has been a major goal of neo-Piagetian theories of cognitive development. Halford's [10] research has led to the
48 conclusion that the best metric for processing capacity is the complexity of relations that can be processed in parallel.
49   In a typical Piagetian class inclusion task, children are shown a collection of objects (e.g., wooden beads), most of
50 which are of one colour (e.g., red) and the rest of another colour (e.g., white). Children are asked if there are more red
51 beads or more wooden beads and are credited with class inclusion if they indicate that there are more wooden beads
52 because the red beads are included in the total class of wooden beads. Under the assumption that each simultaneous
53 value assignment requires a unit of capacity, the operation of class inclusion would require a minimum of 3 such units
54 that means a memory span of 3. It was shown by Humphreys et al. [11] that a total score on 27 Piagetian tasks was very
55 highly correlated ($r = 0.88$) with the 14-item Wechsler IQ test. From only 13 Piagetian tasks Humphreys et al. could
56 form a test that is an excellent measure of general cognitive ability in its own right but can also add to the information
57 furnished by Wechsler Verbal and Performance IQs and academic achievement. Piagetian tasks and ordinary IQ test
58 item differ only that in Piagetian tasks this minimum of memory span to solve the task is known, in ordinary tests not or
59 not explicitly.
60   Pascual-Leone understands memory span as the maximum of discrete and equal energy units (i.e. quanta) which
61 every subject has at his disposal. In the first step of Pascual-Leone's experimental procedure all subjects learned a small
62 repertoire of stimulus-response units. The responses were overlearned motor behaviours such as: raise-the-hand, hit-
63 the-basket, clap-hands, etc. If a subject has a memory span of 5 and it has to keep in mind a memory set of 5 elements,
64 he cannot arrange element 1 corresponding to span or attention space 1, element 2 to span 2 and so on. This will be
65 impossible. Because access to chunks in working memory is random, the available energy quanta are not distinguishable
66 and have to be defined as bosons (i.e. indistinguishable quanta). By applying the Bose–Einstein occupancy model of
67 combinatorics to his learning experiments with children of different age, Pascual-Leone obtained a very good agreement
68 between empirical probabilities and Bose–Einstein predicted theoretical probabilities. Weiss [12] calculated from
69 Pascual-Leone's sample of 11.8-year-olds a mean information entropy H of 86.4 bits. A mean IQ of 119 for 11.8-year-
70 olds corresponds in performance to an adult IQ of 102 for about 40-year-olds. In tables of IQ test results edited by Lehrl
71 et al. [13] and based on concepts of information theory (see below), we read for this age and IQ 102 a short-term
72 memory storage capacity of 84 bits. Two approaches with seemingly completely differing theoretical starting points lead
73 on the absolute scale of information entropy to practically the same result. For Pascual-Leone's data the latter result
74 was even obtained after applying quantum mechanics twice in series, for calculating Bose–Einstein statistics and in-
75 formation entropy.
76   The variance of the Bose–Einstein distribution equals $m^2 + m$, where m reflects the granularity of the energy due to
77 Einstein's photons (cited from [14]). If we set the variance 1 and $m = x$, we get $x^2 + x = 1$. The solution of this equation
78 is ϕ ($= \sqrt{5} - 1/2 = 0.618033$), the golden mean. [1] Its inverse $1/\phi = \Phi$ (also called the golden ratio, the golden number,
79 the golden section or the divine proportion) has the property $1 + \Phi = \Phi^2$. Therefore the double geometric Φ-series:

$$\ldots, 1/\Phi^2, 1/\Phi, 1, \Phi, \Phi^2, \Phi^3, \ldots$$

81 has the properties,

$$\ldots, 1/\Phi^2 + 1/\Phi = 1, 1/\Phi + 1 = \Phi, 1 + \Phi = \Phi^2, \ldots \tag{1}$$

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[1] Some authors call its inverse Φ ($= \sqrt{5} + 1/2 = 1.618033$) the golden mean. We hope this will cause no confusion.

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83 and is thus a Fibonacci series. It is the only geometric series that is also a Fibonacci series. Essential is the fact that the
84 fractional parts 0.618033… of ϕ, $1/\phi$, and $1/\phi + 1 = \Phi^2$ are identical. The title chosen by us refers to this golden mean
85 in the broader sense.

86   Forces are now recognised as resulting from the exchange of huge numbers of discrete particles, or information
87 patterns called vector bosons, which are exchanged between two or more particle information patterns. The absorption
88 of a vector boson information pattern changes the internal oscillation state of a particle, and causes an impulse of
89 motion to occur along a particular direction. This turns out to be the quantum origin of all forces. Therefore, forces can
90 be thought of being digital rather than analogue.

91   In 2001 Bianconi and Barabási [15] discovered that not only neural networks but all evolving networks, including the
92 World Wide Web and business networks, can be mapped into an equilibrium Bose gas, where nodes correspond to
93 energy levels and links represent particles. Still unaware of the research by Pascual-Leone, for these network researchers
94 this correspondence between network dynamics and a Bose gas was highly unexpected [16].

95 3. The information entropy of working memory capacity

96   Shannon's information entropy H is the logarithm of the number of microstates or patterns consistent with our
97 information. To reduce the information to this scalar measure (evaluated in bits of accuracy gained) is to reduce the
98 form to its topological complexity. Each global state of a network can be assigned a single number, called the energy of
99 that state, i.e. the differences in the log probabilities, and hence information entropy of two global states (thoughts) is
100 just their energy difference. By extension of Shannon's concept of channel capacity [17], in 1959 Frank [18] had claimed
101 that cognitive performance to be limited by the channel capacity of short-term memory. He argued that the capacity H
102 of short-term memory (measured in bits of information) is the product of the processing speed S of information flow (in
103 bits per second) and the duration time D (in seconds) of information in short-term memory absent rehearsal. Hence

$$H(\text{bits}) = S(\text{bits/s})D(\text{s}). \tag{2}$$

105 According to Frank the mean channel capacity follows a lognormal distribution [19], where 140 bits correspond to IQ
106 130, 105 bits to IQ 112, and 70 bits to IQ 92.

107   The first experimental approach to determine mental processing speed in bits per second was accomplished by
108 Naylor [20]. His method of testing enabled the subjects to present to themselves a stimulus which remained as long as
109 they kept a finely balanced switch depressed. The stimuli were digits between 1 and 9 or numbers between 1 and 32
110 presented singly or in groups of two, three, four, or five. By this procedure the time was measured until the signs were
111 perceived by the subjects. The information content of one digit of the repertoire of nine possibilities was $2^{3.17} = 9$. That
112 is, 3.17 bits. Recognition of one of the 32 possibilities ($= 2^5$) was equal to 5 bits. Thus, Naylor measured not only the
113 time between stimulus and reaction but also the amount of stimulus information. This is the prerequisite for the more
114 striking observation by Lehrl and Fischer [21], that the results (in bits/s) are numerically equal although the repertoires
115 if signs differ. The measurement of stimuli and reaction in terms of the information unit (the bit) and physical time will
116 only reveal properties of the subject if the information content of the objective repertoire agrees with that of the
117 subjective repertoire. When a repertoire of signs (such as letters, digits or chunks) is overlearned, independently pre-
118 sented signs, whether of sense or nonsense in common usage, have the same objective as subjective information.

119   Instead of applying one of the elementary cognitive tasks already mentioned, Lehrl et al. operationalised Frank's
120 concept of short-term memory storage capacity (in bits) by testing memory span and reading rate. The subject is simply
121 asked to read a series of mixed up letters in an undertone as quickly as possible. As soon as the subject begins to speak,
122 the stopwatch is started. The time from the first to the last spoken letter is measured. It should be documented in tenths
123 of a second, e.g., 7.3 s. When evaluating the raw scores it must be remembered that a subject can only perform full
124 binary decisions. Therefore, the recognition of a letter out of the repertoire of 27 letters, which theoretically has an
125 information content of 4.7 bits ($27 = 2^{4.7}$) needs five binary decisions. Since each letter contains 5 bits of information,
126 the 20 letters contain 100 bits. This is divided by the time of reading to obtain the amount of information processed in a
127 second S (bits/s). For example, if the best time of a subject is 7.3 s, then $S = 100/7.3$ (bits/s) $= 13.7$ bits/s. By stan-
128 dardising letter reading on adults, normative data are available (see Table 1; column mental speed).

129   Forward memory span D can be predicted on the basis of the number of simple words which the subject can read out
130 in 1.8 s. Regardless of the number of syllables, any subject in an empirical investigation by Baddeley et al. [22] was able
131 to recall as many words as he could read in 1.8 s. This result can easily be confirmed by the normative data from Lehrl
132 et al. For example, for IQ 100 holds: The 20 letters of their reading task are read in 6.6 s; D (memory span) corresponds
133 to 5.4. Now we can calculate $x = 6.6 \text{ s} \times 5.4/20 = 1.8\text{s}$. Hence, span and processing rate are both measures of the same

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Table 1
Memory span (corresponding to the number of an EEG harmonic), frequency of EEG harmonics and mental speed and their relationships with information entropy, power density of short-term memory storage capacity, and IQ

a	b		c	d	e	f	g	h
Memory span n	EEG harmonic			Mental speed $kT \ln 2$		Information entropy (bit, $kT \ln 2$)	Power density $E = n^2 2\Phi kT \ln 2$	IQ
	f (Hz)	$E = nfkT \ln 2$		bits/s	bit/s			
9	29	261		29	261	234	262	146
8	23	184		25	200	190	207	139
7	21	147		24	168	154	159	133
6	17	102		18	108	112	116	118
5	13	65		14	70	68	81	93
4	10	40		10	40	42	52	78
3	6.5	19.5		9	27	36	29	76
2	5	10		3	6	11	13	65
1	–	–		–	–	–	3	–

Column b: Empirical data from Liberson [26].
Column c: Product of column b × n.
Column e: Product of column b × n.
Columns a, d, f and h: Empirical psychometric data from Lehrl et al. [13].
Their sample size for standardising the test was 672 subjects.
Column g is purely theoretical.

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134 working memory system [23]. The greater the memory span, the faster the processing rate. The time required to process
135 a full memory load is a constant, independent of the type of material stored.
136   The overall importance of reading speed in everyday life and as an indicator of processing speed is obvious. With
137 increasing age, children name familiar objects more rapidly, and these naming times are related to reading ability.
138 Greater memory capacity is associated with greater reading recognition skill, and the same comprehension processes
139 underlie both reading and auding. The fastest rate that individuals can successfully operate their reading and auding
140 rate is limited by their thinking rate. Consequently, there is an inverse relationship between the length of words and
141 their frequencies of usage. Because words are stored in neural networks, the discovery by Bianconi and Barabási [16]
142 reveals the deeper meaning of Zipf's and Pareto's power law [24] by which the size of the vocabulary of a given in-
143 dividual can be understood as a function of his memory span n.

4. Memory span and EEG

145   During the last decades a number of authors have claimed not only correlations between memory span and mental
146 speed, but also with electrophysiological variables of the EEG. In 1935, Gibbs et al. [25] had already documented that
147 patients (sample size was 55) with petit mal epilepsy show, in all cases during seizures, an outburst of spike waves of
148 great amplitude at a frequency of about 3 s^{-1}. The fact that such seizures can be aborted using brief stimuli is very
149 suggestive of an underlying multistable dynamical system. This finding is part of our confirmed knowledge and can be
150 read in every textbook on EEG or epilepsy. From this Liberson [26] had drawn the conclusion that all significant
151 channels in EEG could be n multiples of one fundamental frequency of about 3.3 Hz. According to his empirical data
152 the number of these multiples (harmonics) is nine as the maximum of memory span (see Table 1). Assuming these
153 numbers 1–9 to be quanta of action (as Pascual-Leone did), we again obtain a relationship between the classical for-
154 mulae of quantum statistics and empirical results of both EEG and psychometric research.
155   Assuming the numbers 1–9 of memory span to be equivalent of harmonics in the sense of wave theory, the power
156 spectral density E is given by the eigenstate energy–frequency relationship $E = nf(kT \ln 2)$, where f is frequency. Ac-
157 cording to thermodynamics, the measurement of 1 bit of information entropy [27] requires a minimum energy of 1
158 $kT \ln 2$, where k is Boltzmann's constant and T is absolute temperature. During the duration of 1 perceptual moment 1
159 bit of information is processed per harmonic. That means that 1 break of symmetry and 1 phase reversal after each
160 zero-crossing of an EEG wave corresponds with a possible 1 bit decision between two alternatives. Consequently, each
161 degree of freedom and of translation (this refers to mathematical group theory [28] underlying both mental rotation and
162 quantum mechanics) corresponds to an energy of $kT/2$ or its macroscopic analogon.
163   Because the frequency of EEG harmonics can be expressed as $n2\Phi$ Hz, for the expected latencies of harmonics
164 follows 1000 ms/$n2\Phi$ and for power density follows $E = \sum n2\Phi(kT \ln 2)$. The physical term power is appropriate be-
165 cause it is a measure of the ability of waves at frequency f to do work. The power spectrum to the EEG describes the
166 total variance in the amplitudes due to the integer multiples of the fundamental frequency (i.e. the first harmonic
167 $1 \times 2\Phi$). In order to calculate power density in this way, the waveform must be squared and then integrated for the
168 duration of its impulse response, i.e. the duration of the transient of one complete wave packet containing all the
169 harmonics of the memory span of a given subject.
170   The relationships in Table 1 are further supported by data from Bennett [29], who reanalysed the Ertl and Schafer
171 [30] findings of a correlation between IQ and latencies of EEG evoked potential components. Bennett (confirmed by
172 Flinn [31]) accomplished a Fourier transformation of the original data and found that high IQ subjects (IQ above 123)
173 go through 20 or more perceptual moments per second, low IQ subjects (IQ below 75) only through 8 moments or even
174 less (compare Table 1, columns b and d). This striking parallelism between EEG results and channel capacity, measured
175 with mental tests, is emphasised by results from Harwood and Naylor [32]. About 42 young university students had a
176 mean channel capacity of 21.4 bit/s; 105 "average normal" adults who were 60–69 years old performed 14.2 bit/s; the
177 age group of 70–79 years (sample size was 67) achieved 12.9 bit/s; and 13 subjects being 80 years and older 10.2 bit/s,
178 thus reflecting the usual decline of mental performance of old aged people. Pure coincidence in this parallelism of
179 channel capacity and EEG frequencies (compare Table 1) seems impossible: neither Liberson nor Lehrl, neither Bennett
180 nor Naylor nor Pascual-Leone knew anything about the results and theories of the others.
181   Higher IQ subjects have not only a higher memory span, but consequently also more complex waveforms of EEG
182 than lower IQ subjects. The most extreme compression of information is represented by the eigenvalues of the power
183 spectrum. There are as much eigenvalues of a spectrum as are harmonics [33]. Already in 1959 Burch (cited from [34])
184 had found that "the parameters ... of the power spectral density ... can be estimated in a completely adequate way

185 without the necessity of performing squaring and integrating operations but simply by counting the zero crossings.''
186 The number of zero-crosses up to the P300 of evoked potentials is *the upper bound* of the memory span of an individual.
187   In such a way memory span has to be understood as the quantum of action of thought. In fact, these quanta of
188 action represent macroscopic ordered states in the sense of quantum mechanics. Empirical analysis shows that Lib-
189 erson's fundamental is lower than 3.3 Hz and in the range between 3.1 and 3.3 Hz. The reliability of the empirical data
190 allows no more precise calculation. Nevertheless, it could be imagined that a numerical constant underlies the har-
191 monics of the EEG, enabling brain waves to process information in the most efficient way. Because Hz is a manmade
192 measure, depended on the definition of the second, an exact solution seems to be mere numerology and no scientific
193 argument. Despite this, the congruence between multiples of memory span and multiples of a fundamental brain wave is
194 the first important discovery derived from Table 1, the precise size of the fundamental seems to be a problem of second
195 order.
196   From technical applications we know that an array consists of equally spaced sensors making measurements at
197 discrete intervals [35]. Only under this condition frequency bands and wave number can be detected in the spatio-
198 temporal domain. If a travelling wave is spatially sampled using such a discrete array of sensors, an estimate of the wave
199 is obtained by appropriately delaying or advancing the signals on each of the channels and summing the results.
200 Therefore the idea that brain architecture and neural networks, respectively, should be understood in terms of se-
201 quences of delaying chains and matched filters facilitating run-length coding is not a new one [36].

202 **5. The golden mean as resonant frequency**

203   It is a psychoacoustic fact, known as octave equivalence [37] that all known musical cultures consider a tone twice
204 the frequency of another to be, in some sense, the same tone as the other (only higher). The point of resonance,
205 corresponding to the eigenvalues and zero-crossings of a wave packet (wavelet), is not the frequency of its fundamental,
206 but half of its frequency. If we assume the fundamental to be twice the golden mean [38] Φ, that means $2 \times 1618 = 3.236$
207 Hz, a point of resonance at $\Phi = 1.618$ Hz follows.
208   Datta [39] showed how a sense of time and evolution is intrinsically defined by the infinite continued fraction of the
209 golden mean and its inverse. The real number set gets replaced by an extended physical set, each element of which is
210 endowed with an equivalence class of infinitesimally separated neighbours in the form of random fluctuations. Time
211 thereby undergoes random inversions generating well-defined random scales, thus allowing a dynamical system to
212 evolve self similarly over the set of multiple scales. These random fluctuation generate $1/f$ noise, which is one of the
213 footprints of complexity at the critical border between predictable periodic behaviour and chaos. Datta was unaware of
214 some empirical results already supporting his theory. The distribution of the time elapsed between two consecutives
215 spikes in the firing response of visual cortex neurons has been studied in cat [40] and macaque [41]. The distribution of
216 time intervals clearly follows a power law over several orders of magnitude. In both experiments the exponent of the
217 time separating two firings was roughly equal to 1.60 ($\approx \Phi$).
218   According to Datta [39], it seems reasonable to assume that time may change from $t-$ to $t+$, not only with the usual
219 arrow, but also instantaneously by an inversion. The definition of time inversion has an in-built uncertainty, thus el-
220 evating time itself to the status of a random variable. However, in the midst of all fluctuations there exists the golden
221 mean equation $\phi(\phi)^2 + \phi(\phi) = 1$. In his theoretical approach, to understand all physical constants as random sample
222 of independent numbers following a $1/x$ probability law, Frieden came to the conclusion that the median value of all
223 constants ought to be precisely 1, and he stated: ''Why the value 1 should have this significance is a mystery. The
224 probability density function is invariant to a change of units. Therefore, the median of the constants is 1 *independent of*
225 *the choice of units*. This gives added strength to the result and to the mystery'' [42, p. 226]. ''This result holds inde-
226 pendent of units, inversion and combination, since the $1/x$ law itself is invariant under these choices. Therefore, the
227 median value of 1 is a *physical effect*'' [42, p. 233]. We see no other solution to this mystery as the golden mean equation
228 (for the even deeper relationship between the golden mean and the prime number distribution see, for example, [43]). In
229 other words: For any observer there is no simple 1 in the world but only the golden mean as the only point of certainty
230 of any measurement. Only if $K = 1$, the point of the nearest neighbour coupling strength K of the block-spin technique
231 remains for ever on the ridgeline of a hyperbolic paraboloid cited from [38, p. 156].
232   In 1995 Gilden et al. [44] asked subjects to reproduce m times a given time interval, chosen between 0.3 and 10 s, by
233 pushing a button on a keyboard. The error was then recorded, interpreted as a time series and its power spectrum
234 computed. The resulting power spectrum behaved like $1/f^y$ with y about 1.
235   This insight that the measurement of any physical quantity and quality is based on repetitions of the golden mean,
236 opens an astounding variety of possibilities to encode and decode information in the most efficient way. With this

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237 property the brain can use simultaneously the powers of the golden mean and the infinite Fibonacci word [45] (syn-
238 onymously called the golden string, the golden sequence, or the rabbit sequence) for coding and classifying. Every
239 positive integer can be a sum of Fibonacci numbers; it can also be understood as a finite sum of positive and negative
240 powers of the golden mean. A binomial graph of a memory span n has n distinct eigenvalues and these are powers of the
241 golden mean. The number of closed walks of length k in the binomial graph is equal to the nth power of the $(k + 1)$-st
242 Fibonacci number [46]. The total number of closed walks of length k within memory is the nth power of the kth Lucas
243 number.
244   Lifeforms maximise their adaptive capacities by entering the region of complexity on the edge of chaos. From the
245 period doubling route of chaos it turns out that when $R = 2\Phi = 2 \times 1.618 = 3.236$ one gets a super-stable period with
246 two orbits, producing the first island of stability. Thus, the quasi periodic Φ toroid geometry is the most stable under
247 perturbation. The orbit is of the lowest period possible (being two) and therefore crucially consumes the least energy to
248 maintain. Bands of order in the Feigenbaum diagram occur at a fixed scaling mean, where all bifurcations, representing
249 the length $w1 = (\Phi - 1)/2$, are positioned at $a = 2\Phi$ [47]. This is how Φ is embedded within dynamical systems, as a
250 universal binary shift operator, or primary eigenfunction. All constants so derived have to be eigenvalues of this op-
251 erator (think of resonances and harmonics).
252   The existence of a Fibonacci series and the convergence of the ratio of the winding numbers of an orbit towards
253 $\phi = \sqrt{5} - 1/2 = 1/\Phi$ in a Hamiltonian system is a numerically well-known phenomenon of physics. The mathematical
254 foundation and proof of this phenomenon is the essence of the theorem of Kolmogorov, Arnold and Moder. From this
255 theorem follows, too, that the golden mean, which is the most irrational number, must give the most stable orbit.
256 Irrational values of the winding number correspond to an uncountable set of zero measures of values—in other words
257 the irrationals are squeezed into a Cantor dust [48].
258   A slide-rule computes products because the marks on the sliding ruler correspond to logarithms, and adding two
259 logarithms is equivalent to multiplying the corresponding pair of numbers. Also the Fibonacci and Lucas numbers can
260 be understood like the markings on a ruler that is recursively divided into golden mean pieces. By using powers of the
261 golden mean any multiplication can be reduced to an addition. The golden mean is the mean of the sides of a rectangle
262 circumscribed about a logarithmic spiral, too. Logarithmic spirals are, like fractals, self-similar at all scales. Therefore
263 our brain performs visual computation at several scales (demagnifications of the image) and compares the results [49].
264 With a sampling algorithm, based on Fibonaccis and phyllotaxis [50], even coloured images can be quantized and
265 processed [51].
266   If we draw a line $y = \Phi x$ on a graph (i.e. a line whose gradient is Φ) there we can see directly the binary expression of
267 the Fibonacci sequence known as the infinite Fibonacci word. Where the Φ line crosses a horizontal grid line (imagine
268 the discrete columns of the brain) we write 1 by the line and where the Φ line crosses a vertical line we write a 0. As we
269 travel along the Φ line from the origin we meet a sequence of 1s and 0s. The 1s in the Fibonacci string 1011010110...
270 occur at positions given by the spectrum of Φ and only at those positions [52]. Trajectories of dynamic systems whose
271 phase spaces have a negative curvature everywhere can be completely characterised by such a discrete sequence of 0s
272 and 1s. The self-similar Fibonacci string reproduces itself upon reverse mapping or decimation, both fundamental
273 properties from the point of information storage and retrieval. After decimation by a factor of the golden mean every
274 unit in the original lattice coincides precisely with a unit in the compressed lattice. From the point of view of renor-
275 malization theories of physics, the decimation process is the complement of deflation or block renaming. Any 1 in the
276 Fibonacci string forces an infinite number of symbols in a characteristic quasi-periodic pattern. For any such Sturmian
277 sequence the topological structure completely determines all the Markov approximations. It means that only one
278 ergodic measure is compatible with the topological structure.
279   For computer science the Fibonacci string is no newcomer [53]. Processing of strings of symbols and string rewriting
280 is the most fundamental and the most common form of computer processing: every computer instruction is a string, and
281 every piece of data processed by these instructions is a string. A repetition in a string is a word of the form 11 or 00,
282 called a square. The frequency of such squares is a function of the logarithm of the golden mean [54].
283   Since the fabrication of semiconductor superlattices arranged according to the Fibonacci and other sequences, there
284 has been a growing interest in their electronic properties. When a homogeneous electric field is applied perpendicular to
285 the layer plan, electronic states become localised and the energy spectrum consists of a Wannier–Stark-ladder, char-
286 acterised by a sequence of metastable states of resonance separated by equal energy intervals. An initial Gaussian wave
287 packet is filtered selectively when passing through the superlattice. This means that only those components of the wave
288 packet whose wave number belong to the allowed harmonics of the fractal-like energy spectrum can propagate over the
289 lattice. Diez et al. [55] discuss therefore, aside from the possibility of building filter-like devices, designed with Fibonacci
290 or a binary quasi-periodic sequence according to the desired application, the possibility that such a kind of system can
291 be used in processing information. Surely, the insight that our brain uses very similar physical and mathematical

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292 properties will accelerate technical progress in this area. Cuesta and Satija [56], studying empirically Fibonacci lattices
293 with defects, even found: "Novel result of our studies is the relationship between the resonant states and the states
294 where the energy bands cross. We show that the resonant states are fully transmitting states in the quasi-periodic limit
295 and are described by the wave functions that are related to the harmonics of the sine wave with fundamental Bloch
296 number equal to the golden mean." Bloch waves are known as the most important effect due to the discrete lattice
297 translational symmetry. This arises because the Hamiltonian must commute with the translational operator for any
298 discrete integer lattice translation. The wave function can be represented as the product of a plane wave with a periodic
299 function. The translational invariance of the wave function is of utmost importance. This basically indicates that all
300 information about the system is stored within an excited subset of the system; the rest of the non-resonant information
301 is redundant at this very moment. If we stress the analogy between waves in quasi-periodic lattices and the phenomenon
302 of memory span in our brain, this seems to be an especially important point. We confess to have the vision of multilayer
303 hierarchical binary or Fibonacci semiconductor superlattices simulating the calculating and classifying capabilities of
304 our brain, far surpassing the brain by the higher speed of the technical application.
305   It is already well-known among electrical engineers [57] that the characteristic impedance of an electrical ladder
306 network, which is needed for an error-free connection, has to be a function of the golden mean. Even the sound by any
307 stereo system depends on the purity of the audio signal it produces. Each strand in a cable has its own beat. When the
308 cable linking all components together imparts its own sound, the audio signal is corrupted. George Cardas received US
309 Patent Number 4,628,151 for creating Golden Mean Stranding Audio Cable. Individual strands are arranged so each
310 strand is coupled to another, whose note or beat is irrational with its own, thus nulling interstrand resonance.

311 6. The universe as a world of numbers

312   El Naschie [58,60,61] and others developed for the fundamental question of time reversibility the notion of a
313 Cantorian space–time (compare the idea of Cantor coding by Tsuda [59]). What is really remarkable of this Cantorian
314 space–time is that applying all the probabilistic necessary laws, the values of the Hausdorff dimension are intrinsically
315 linked to the golden mean and its successive powers. The correlated fluctuations of the fractal space–time are analogous
316 to the Bose–Einstein condensation phenomenon. The polynomial roots of higher order Fibonaccis, scaling a quasi-
317 periodic hierarchy, are based on golden mean powers.
318   There can be no doubt that our brain uses for computing inherent and inborn properties of the physical universe. We
319 have or learn into the neural network of our brains the relationships between external stimuli, the integer powers of the
320 golden mean, the Fibonacci word and Lucas numbers, and we are probably able to use the relationships between the
321 Beatty sequences of e, π and Φ, and we use hundreds of similar relationships (many of them may still be undiscovered
322 by contemporary mathematics) between numbers for encoding and decoding information simultaneously and uncon-
323 sciously by wavelets. A genius like Ramanujan gave us some closed fraction formulae which contain π, e and Φ all
324 together in a single equation. Together with Euler's famous formula $e^{i\pi} + 1 = 0$ for the unit circle we all understand in
325 our subconsciousness these irrational numbers as rules for superposition and time reversal by folding, symmetry
326 breaking and compactification. By raising $\Phi = 1 + \phi$ to the third power , we get the Hausdorff dimension of Cantorian
327 space–time of El Naschie [61] $(1 + \phi)^3 = 2 + \sqrt{5} = 4 + \phi^3 = 1 + \phi/1 - \phi = 1/\phi^3 = 4.236$, which plays also a profound
328 role in knot theory, von Neumann's algebra, quasi-crystals and non-commutative geometry. But who could expect such
329 a result and such connections on the basis of deceptively simple mathematics?
330   Quantum mechanics seems to require the quantization of all physical quantities on the small scale, yet space and
331 time are still treated in most cases as a classical space–time continuum, where there are an infinite number of space
332 points between any two given locations, no matter how close. Therefore many physicists agree that the current set of
333 fundamental physical laws is incomplete. Because Hz, oscillations per second, is superficially seen only a man-made
334 measure, this seems to be the weakest point of our line of reasoning. Behind the definition of the second is the velocity of
335 light ($c = 299\,792\,458$ m/s), which is the constant on which size all other physical constants depend upon and hence
336 represents the inherent speed limit that any particle information pattern is able to achieve. In this system of present-day
337 constants the Planck length has the value 1.6160×10^{-35} m (standard uncertainty 0.0012×10^{-35} m). If we fix instead the
338 Planck length at the value of the golden mean at 1.6180×10^{-35} m and recalculate consequently all other physical
339 variables, this means for the numerical size of the second only a trivial correction not relevant for our argument.
340   Indeed, there is a growing minority of scholars who understand the world as something like a cellular automaton
341 running with and counting numbers. The numerical state of all the cells, everywhere, changes at a regular synchronised
342 interval called a clock cycle. The universal cellular automaton seems to be capable of updating its entire memory in a
343 single clock cycle, which according to Occam's razor could be nothing else than the Planck time as the relation between

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344 the velocity of light and the Planck length, the latter fixed at the value of the golden mean. If we, for example, look into
345 Wolfram's "A New Kind of Science" we see that this Model 3 and Model 4 automata are full of Pascal triangles. Behind
346 such a triangle are always the Fibonaccis and hence the golden mean. That means that by encoding and decoding the
347 information of such and any automaton or system no other wave could be more optimal than a wavelet containing the
348 golden mean itself. The quantization of time simply represents the number of regular clock cycles elapsed between two
349 events and all changes that occur must occur as localised changes. At the lowest level our brain seems to be utterly
350 simple, deterministic and mathematical in nature. Despite this, we can never read out the numeric state of any brain in a
351 foreseeable future. We can only infer this type of information by observing larger scale patterns as it is the phenomenon
352 of memory span.

353 **7. Conclusion**

354   Our paper is not an end, but a beginning. A new theory, in fact any theory must end in one way or another by
355 confirming what we all know to be the case, namely that space–time forms an effectively four-dimensional manifold
356 compatible with the space–time of classical physics as well as general relativity. In addition, if this theory is to be
357 regarded as something new then it must be quantized in much the same way as in the Planck theory of radiation, only
358 here it is space and time themselves that are quantized. None the less, at the end we must still recover our ordinary
359 space–time, where measurements are being made and out of which is no escape. Since 1990 El Naschie [60] is building
360 such a theory, in which the golden mean as a universal constant is playing a fundamental role and in which context our
361 meta-analysis of empirical results is not a logical chain of incredible wonders, but a logical consequence of the observed
362 fact that the electric and other charges of particles are simple rational multiples, theoretically existing in a universe
363 where time is fully spatialised and nothing more than a random Cantor set fluctuating with a golden mean Hausdorff
364 dimension. In order to understand our brain, there is no other way than to come to a deeper understanding of the world
365 around us.

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