# The Spatial Metric of Brain Underlying the Temporal Metric of EEG and Thought

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With 1 Table

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### Abstract

Multiplying memory span by mental speed, we obtain the information entropy of short-term memory capacity, which is rate-limiting for cognitive functions and corresponds with EEG power spectral density. From psychometric and EEG data follows a fundamental of about 3.14 Hz. The number of harmonics (n = 1, 2, ..., 9) is identical with memory span, and the eigenvalues of the EEG impulse response are represented by the zero-crossings up to the convolved fundamental, the P300. The difference T of 4.42 ms between the 8th and the 9th harmonic is the smallest time window of conscious information processing.

Shannon's sampling theorem allows the replacement of any band-limited signal by m discrete sequences of T without loss of any information. Brain architecture can be understood in terms of sequences of delaying chains. Acting as a wavefront tracking array, scaled in relation of m and in such a way also expressing the metric of eigenvalues, widths of orientation columns match with phase reversal after a zero-crossing and lengths of dendritic trees with runlength of travelling harmonics.

Key words: EEG spectral density, impulse response, brain architecture, run-length coding

#### 1. Introduction

A slide-rule computes products because the marks on the sliding rules correspond to logarithms, and adding 2 logarithms is equivalent to multiplying the corresponding pair of numbers. Thus, the physical system carries out a computation by virtue of its relation to a more abstract algorithm. Our brains classify objects and compute. Each healthy individual is able to remember thousands of words, faces, and impressions within a few ms and is able to answer in a time range of less than 1 s, whether a word was already stored in memory or is entirely new. What we need to understand is the metric and the mechanism of our brain machinery (BAŞAR 1988).

Can one hear the shape of a drum? In order to answer this question, KAC (1966) asks for the energy in the frequency interval df. To this end, he calculates the number of harmonics which lie between the frequencies f and df and multiplies this number by the energy which belongs to

the frequency f, and which according to the theory of quantum mechanics is the same for all frequencies. By solving the eigenvalue problem of the wave equation, KAC is able to state that one cannot only hear the area of a reflecting surface, its volume and circumference, but also—and this seems to be the most exciting result in view of modern network theories (see, for a review, Cowan and Sharp [1988])—the connectivity of paths of an irregular shaped network. If the brain waves had the possibility to measure and hence to "know" the eigenvalues of an information amount, they would have nearly perfect access to information and—in terms of communication theory—perform nearly perfect band-limited processing.

The metric of time underlying the eigenvalues of EEG brain waves could be the metric that generates a relationship between energy, information, and distance of anatomical structures.

Therefore, at first, we will outline some relationships between well-confirmed psychometric and psychophysiological empirical facts (compare also Eysenck [1986, 1987]) and EEG spectral density.

## 2. The information entropy of short-term memory storage capacity

SHANNON'S (1948) information entropy H is the logarithm of the number of microstates consistent with our information. And THOM (1975; p. 157) stated:

To reduce the information to this scalar measure (evaluated in bits of accuracy gained) is to reduce the form to its topological complexity.

Each global state of a network can be assigned a single number (GIBBS 1981), called the energy of that state, i.e. the differences in the log probabilities, and hence information entropy of 2 global states ("thoughts") is just their energy difference.

By extension of Shannon's concept of channel capacity, Frank (1960) had introduced the concept of short-term-memory (STM) storage capacity as the rate-limiting variable of all cognitive functions. Frank operationalized the information entropy H [bit] as the duration I [s] of memory span multiplied by mental processing speed V [bit/s] that is

$$H = I \times V. \tag{1}$$

Processing speed can, in psychometric test batteries, be operationalised by measuring choice reaction time or speed of mental rotation, through reading rates (Lehrl et al. 1980), scanning information in STM, inspection time, and time to escape masking (Geissler 1987 a, b).

Short-term memory span is defined as the number of items (digits, letters, chunks) that a person can attend to simultaneously or retain together in immediate working memory. One of the well-established facts of psychology is that human STM is limited to retention of  $7\pm2$  items. The school of neoPiagetian psychology has claimed memory span to be the most important human limitation in reasoning and problem solving. In his pioneering research Pascual-Leone (1970) understands memory span as the maximum of discrete indistinguishable energy units which every subject has at his disposal (for more details see Weiss [1986]).

## 3. Relationships between EEG parameters and psychometric results

Already GIBBS et al. (1935) had documented that patients with petit mal epilepsy show in all cases during seizures an outburst of spike waves of great amplitude at a frequency of about 3/s. From this in 1938 LIBERSON (see 1985) had drawn the conclusion that all significant channels in EEG are *n* multiples of *one* frequency of about 3.3 Hz. According to his empirical data the number of these multiples is 9 as the maximum of memory span (see Table 1).

Table 1. Memory span (corresponding to the number of an EEG harmonic), frequency of EEG harmonics, channel capacity (i.e. perceptual moments/s) and their relationships with information entropy, power density of short-term memory storage capacity, latencies of harmonics and IQ

a	b	С	d	e	f	g	h
Memory span	EEG harmonic	9	Channel capacity	Information entropy	Power density	Latency	IQ
n	f	E = nf	bit/s	bit	$E=n^2\pi$	$\frac{1000 \text{ ms}}{n\pi}$	45
	[Hz]	$[kT \times ln2]$		$[kT \times ln2]$	$[kT \times ln2]$	[ms]	
9	29	261	29	230	254	35.37	144
8	23	184	25	181	201	39.79	139
7	21	147	24	162	154	45.48	135
6	17	102	18	109	113	53.05	118
5	13	65	14	67	79	63.65	93
4	10	40	10	44	50	79.55	78
3	6.5	19.5	9	30	28	106.16	76
2	5	10	3	11	13	159.24	65
1	•	•	•	•	3	318.31	•

<sup>•</sup> no reliable empirical data

Column b: Empirical data from LIBERSON (1985).

Column c: Product of column b times n.

Columns a, d, e, and h: Empirical psychometric data from Lehrl et al. (1980). — Their sample size for standardising the test was 672 subjects. — Notice, that column e are empirical data and not the product of column d times n. Columns f and g are purely theoretical. However, Liberson (1985) has published similar empirical latency components of event related potentials.

Assuming the numbers 1 to 9 to be harmonics, in accordance with Parseval's theorem (see Båth 1974) the power spectral density E is given by the eigenstate energy-frequency relationship (see Yu 1976)

$$E = nf[kT \times \ln 2], \tag{2}$$

where f is the frequency. According to thermodynamics, the measurement of 1 bit of information entropy requires a minimum energy of  $1 \, kT \times \ln 2$  (SZILARD 1929), where k is BOLTZMANN's constant and T is the absolute temperature.

During the duration of 1 perceptual moment 1 bit of information is processed (Lehrl and Fischer 1988) per harmonic. Each degree of freedom of oscillation corresponds to 1 kT, each degree of freedom of rotation and translation to an energy of kT/2.

Empirical data (Weiss et al. 1986) show that Liberson's (1985) fundamental is lower than 3.3 Hz and in the range between 3.1 and 3.2 Hz, i.e. near 3.14 Hz. Therefore, in the following (as in Table 1), we will write for simplification  $\pi$  Hz. Because frequency can be expressed as  $n\pi$  Hz, for the expected latencies of the harmonics follows 1,000 ms/ $n\pi$  and for power density follows

$$E = n^2 \pi. (3)$$

## 4. The Geissler-Stroud cascade of intrinsic rhythms

Each natural system has a memory whose length is the duration of its impulse reponse, i.e. the duration of the transient of 1 complete wave packet. When a given input signal is fed into a natural system, the memory operates on the input, and the result is the corresponding output signal. The mathematical relationship is that the output is equal to the convolution of the input with the impulse response function. Cross-correlation against a certain wave-form is identical to convolution with the same wave-form reversed in time. For every wave-form, there is a matched filter. The matched filter corresponding to a particular wave-form has an impulse response which is the wave-form itself, reversed in time (CRICK et al. 1981, PRATT et al. 1989).

The event related potentials of the EEG up to the P300 component are the impulse response of the brain. Without doubt, the P300, which plays an unique role in mental chronometry (Başar 1988), is the resonance at the time-reversed fundamental 1,000 ms/1 $\pi$  itself (see Table 1, column g).

Surveying data from a very large number of psychophysiological experiments of other authors or published by Geissler himself and coworkers, Geissler (1987 a, b) drew the conclusion that the temporal architecture of mental processes presupposes an universal constant T of approximately 4.5 ms. His theory assumes that all information processing is based on integer multiples of T and chains of multiples as 2T, 3T,  $2^2T$ ,  $4^2T$  and so on (compare also Stroud [1955] who made a similar assumption, based on a secondary analysis of empirical data by v. Bekesy). Multiples as 4, 8, 16, 20, 24, and 28 are preferred. The upper bound of the highest possible number of such multiples is identical with the storing capacity of STM.

The Geissler-Stroud-theory can also be derived from Table 1 (Weiss 1989). The smallest possible power and time difference, the difference T between the latencies of 9th (35.37 ms) and 8th (39.79 ms) harmonic of the wave packet, is 4.42 ms, itself the 1/72nd part of the fundamental 1,000 ms/1 $\pi$ .

 $9\,T$  is the resonance at the 8th harmonic,  $12\,T$  at the 6th,  $18\,T$  at the 4th,  $24\,T$  at the 3rd,  $28\,T$  at the 2nd. Other important multiples as  $3\,T$ ,  $4\,T$ , and  $10\,T$  can easily be identified as differences between harmonics. All higher multiples of T, which are of importance in GEISSLER's (1987b) theory, as 30, 32, 36, and 64 are differences and resonances at the 2nd harmonic and the fundamental, respectively. And of course, these higher multiples are themselves cascades of multiples.

Sample and hold means to take the samples f(mT) from the input signal f(t) at the time instants t = mT, and holds them constant by delaying them until the next sampling. The probability of the correct identification of a signal by a wave packet (i.e. wavelet; Kronland-Martinet et al. [1987]) is a function of the probability of triggering a determined sequence of frequencies and amplitudes in both space and frequency domains.

If f(t) is a band-limited signal with a spectrum which is  $n\pi$  Hz, then it is determined by its values at the discrete set of points equally spaced at m intervals of T. Shannon's sampling theorem (seen Jerri 1977) allows the replacement of a continuous signal f(t) by a discrete sequence mT of its samples without the loss of any information. If time divided into equal intervals T ms long and if 1 instantaneous sample is taken from each interval in any manner, then a knowledge of the magnitude of each sample plus a knowledge of the instant at which the sample is taken, contains all the information of the signal.

The most extreme compression of information is represented by the eigenvalues (KAC 1966) of the spectrum. These eigenvalues are always multiples of T. Their knowledge allows to transmit and to reproduce any information. There are as much eigenvalues of a spectrum as are harmonics. Each eigenvalue of event related electrocortical potentials is represented by a zero-crossing up to the P300. Up to the P300, the number of zero-crossings is identical with memory span (WEISS 1989). That the number of zero-crossings of an epoch of the EEG up to about 300 ms represents its power spectral density, was already known 1959 to Saltzberg and Burch (see 1971). However, these EEG results were nearly forgotten before the advent of a new brand of signal detection theory, stressing the importance of zero-crossings (Niederjohn et al. 1987, Zeevi et al. 1987, Yuille and Poggio 1988).

From the point of view of Shannon's sampling theorem, the Geissler-Stroud "theory" is only an inevitable consequence and not a theory in its own right.

# 5. The anatomical background of array processing by the brain

Should this make any sense, we cannot evade the following conclusion: There must exist a correspondence between anatomical structure, anatomical array, and the velocity and wave-number of electrocortical waves. Only under this condition an automation of FOURIER processing can be imagined.

From applications, for example in geophysics and radioastronomy, we know that an array consists of equally spaced sensors making measurements at discrete intervals of T ms. Only under this condition frequency bands and wave-number can be detected in the spatiotemporal domain. If a travelling wave is spatially sampled using such a discrete array of sensors, an estimate of the wave is obtained by appropriately delaying or advancing the signals on each of the channels and summing the results (CHILDERS 1977, KUNG 1984).

The idea, that brain architecture could be understood in terms of sequences of delaying chains, is not a new one. Braitenberg (1961) wrote:

It has been pointed out that the organization of the cerebellar cortex becomes understandable if interpreted as a specialization for accurate timing of impulses. Taking into account the well-known relation between diameter of fibres and velocity of conduction, and interpreting the histological connexion ... as an indication of synaptic

connexions, this histology allows one to predict widely varying delays interposed between the input in one fibre system and the arrival of activity at different ... distances from the point of input, while, on the contrary, the other input system (the climbing fibres) should influence ... cells after a fixed, short time delay.

# MAC KAY (1962) added:

If the elements of the network are ... metastable, they can serve as detectors of temporal coincidence between two or more signals. The resolving time (the smallest change in timing that can be detected) can be taken as a quantum in terms of which to estimate the information-storage capacity of a link. Obviously in order to secure a large capacity, one must have delays which are long compared with this resolving time. This sets a penalty in the corresponding latency of response of the network as a whole ... The use of delay lines as network pathways has of course the further advantage that transient storage, for purposes of running analysis and short-term correlation, is automatically provided.

Logical gates by coincidence of latencies of travelling harmonics are also the core of REISS' (1964) arguments, and according to KRONLAND-MARTINET et al. (1987) each harmonic is part of a logarithmic scale of time windows. Most interesting are the analogy between matched filters and brain architecture as outlined by MILEV (1982) and recent applications of such ideas (WAIBEL et al. 1989).

Do the known anatomical facts support such assumptions? In horizontally organized architectonic fields of the human cerebral cortex the apical dendrites of the neuron in a lower stratum were found to be  $3.15~(\sim\pi)$  times as long as that of the upper, and the strata build up a hierarchy with  $\pi$ -fold distances! This discovery by Bok (1959), the central topic of his monography, did never find the attention it now deserves in the light of the metric of the transient EEG impulse response. In a determined time only the higher harmonics reach the upper ramifications of dendritic trees, but both are needed for "deeper thoughts". Bok (1959) also claimed that the length of dendritic sections in binary dendritic trees (i.e. the distance between each branching) seems to follow the general rule 1:2:4:8.

Far more sophisticated and experimentally confirmed is RALL's (1969) cable theory of neurons. This theory interpretes dendritic trees as being electrotonically equivalent to membrane cylinders of lengths L. The discretised time constants have the property

$$\alpha_n = n\pi/L,\tag{4}$$

where the numbers  $\alpha_n^2$  correspond to the eigenvalues of the dendrites. In those regions of the tree where the oscillatory potential has been displaced farthest from the zero line, the rate of return will be initially more rapid than elsewhere and the length of these regions is determined by the eigenvalues, i.e. by the linear combination of n FOURIER coefficients.

Hubel and Wiesel (1977) end up with a view of the cortex as containing a thousands of small machines of vertically organized systems of columns of more or less identical structure. These discrete columns exhibit an amazingly orderly sequence, with many clockwise and counterclockwise steps that add up to total rotation of up to  $180^{\circ}$  (=  $\pi$ ) every 1 mm. Ingber (1985) claimed, this width of columns would match with the time constants of memory span and with EEG parameters.

Interindividual differences in mental performance have their causes in different practice, but also in differences of energy metabolism. Because glucose represents almost all the substrate for cerebral energy metabolism, it is an outstanding event that 3 separate research groups (DE LEON et al. 1983, Chase et al. 1984, RIEGE et al. 1985) report significant

correlations (around 0.60) between regional cerebral glucose metabolism rate and a number of IQ tests, including memory span (WEISS 1987).

This article tries to review and to summarise well-established facts and to avoid speculations. However, it seems not premature to conclude: The brain seems to be like an organ with a hierarchy of several strata of pipes, matching with the run-length of harmonics (Reiss 1964). Moreover, the lattice of pipes and strata are interconnected to a network, acting as a self-conscious whole by array processing and by producing macroscopic ordered states ("thoughts") in analogy to magnetic bubbles (Weiss 1986, Vichnevetsky 1988).

As it seems, the brain as an ideal detector simply measures the energy of the wave-form. No matter what the stimulus is and how the brain behaves, the metric of signal and memory can always be represented and analysed as a superposition of  $n\pi$  states of different energy and their eigenvalues (Kac 1966). Time is the currency (Jones 1976), interconverting energy, information, and spatial distance (Robinson 1982).

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